



Time Series Analysis of Industrial Electricity Consumption in Nigeria Using Harvey Model and Autoregressive Model

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Abstract: In this paper time series modeling and forecasting of industrial electricity consumption in Nigeria is presented. Specifically, Harvey Model and Autoregressive Model, (AR) are used. The data used are obtained from Central Bank of Nigeria (CBN) Statistical Bulletin for industrial electricity consumption ranging from 1979 – 2014. The results shows that Harvey Model has (r^2) = 80.1% and RMSE = 65.2513 whereas Autoregressive Model has (r^2) = 50.1% and RMSE = 71.3985. Obviously, Harvey model has better prediction accuracy than the AR model. The Harvey model was then used to forecast industrial electricity consumption in Nigeria for the next 15 years (from 2015 to 2029). According to the forecast result by the year 2029 the industrial consumption of Nigeria will stand at 539.65 MW/h as against 468.18 MW/h in 2015.

Keywords: Time Series Analysis, Industrial Electricity Consumption, Forecasting, Harvey Model, Autoregressive Model

1. Introduction

Across the globe, heavy dependence on electricity results in high power demand and hence it requires planning of resources of electricity well in advance to ensure a continuous supply of electricity both now and in the future [1-4]. Therefore, modelling and forecasting of electricity is one of the most important aspects of electric utility planning [5-11]. This requires careful measurement and observation of the patterns of electricity use and its prediction into the future.

Studies have shown that industrial power consumption can be related to the level of industrialization and productivity of a nation [12-14]. Also, effective power supply can also boost the economy of a nation. However, over the years, Nigeria has had perennial short fall in power supply [15-17]. Equally the industrial sector has been affected by this poor power supply and that has affected the productivity of the industrial sector. Accordingly, in this study, the focus is on modeling and forecasting the industrial power consumption in Nigeria.

A large variety of mathematical methods and ideas have been used for energy demand forecasting [18-20]. The quality of the demand forecast methods depends significantly on the availability of historical consumption data as well as on the knowledge about the main influence parameters on the energy consumption. These factors also determine the selection of the best suitable forecast tool. Generally there is no 'best' method. Particularly, data from 1979-2014 and two time series models are used for the modelling and forecasting of industrial electricity consumption in Nigeria. The modelling results are interpreted by statistical tests. The focus of the investigation lies in the application of the selected regression models in predicting and forecasting the industrial electricity demand in Nigeria.

2. Theoretical Background

Two (2) time series models, Harvey model [22-24] and Autoregressive model [24] are considered in this paper and the best model is used to forecast the industrial electricity consumption for the period of 15 years (2015 – 2029) [30]. The best model is selected based on statistical prediction

performance values.

2.1. The Harvey Model

Electricity consumption based on Harvey model is generally given as:

$$f(t) = \alpha(1 + \beta e^{\gamma t})^k \quad (1)$$

The Harvey model is based on the simple modified exponential. The proposed model is given as:

$$\begin{aligned} \ln y_t &= \rho \ln Y_{t-1} + \delta + \gamma t + \varepsilon_t \\ t &= 2, \dots, T \end{aligned} \quad (2)$$

Where $\rho = \frac{k-1}{k}$, $\delta = \ln(k\beta\alpha^{\frac{1}{k}}\gamma)$, and ρ , β and γ

are the parameters of the model to be estimated. ε_t is the error term with mean zero and constant variance.

$$\ln y_t = a \ln Y_{t-1} + b + ct + \varepsilon_t \quad (3)$$

$y_t = Y_t - Y_{t-1}$. Substituting for y_t in equation (3) gives;

$$\ln(Y_t - Y_{t-1}) = a \ln(Y_{t-1}) + b + ct + \varepsilon_t \quad (4)$$

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) = a \ln(Y_{t-1}) + b + ct + \varepsilon_t \quad (5)$$

Let $r_t = \ln\left(\frac{Y_t}{Y_{t-1}}\right)$ and $S_t = \ln Y_{t-1}$ then equation (5) becomes:

$$r_t = a S_t + b + ct + \varepsilon \quad (6)$$

The error is

$$\varepsilon_t = \sum_{i=1}^n (r_i - aS_i - b - ct_i) \quad (7)$$

Let u be the sum of squares of the error, then;

$$u = \sum_{i=1}^n (r_i - aS_i - b - ct_i)^2 \quad (8)$$

$$\begin{aligned} \frac{\partial u}{\partial a} &= -2 \left[\sum_{i=1}^n r_i S_i - a \sum_{i=1}^n S_i^2 - b \sum_{i=1}^n S_i - c \sum_{i=1}^n t_i \right] \\ \frac{\partial u}{\partial b} &= -2 \left[\sum_{i=1}^n r_i - a \sum_{i=1}^n S_i - nb - c \sum_{i=1}^n t_i \right] \\ \frac{\partial u}{\partial c} &= -2 \left[\sum_{i=1}^n r_i t_i - a \sum_{i=1}^n S_i t_i - b \sum_{i=1}^n t_i - c \sum_{i=1}^n t_i^2 \right] \end{aligned} \quad (9)$$

If $\frac{\partial u}{\partial a}$, $\frac{\partial u}{\partial b}$ and $\frac{\partial u}{\partial c}$ are set to zero and rearrange the equations will give:

$$\begin{aligned} a \sum_{i=1}^n S_i^2 + b \sum_{i=1}^n S_i + c \sum_{i=1}^n S_i t_i &= \sum_{i=1}^n r_i S_i \\ a \sum_{i=1}^n S_i + nb + c \sum_{i=1}^n t_i &= \sum_{i=1}^n r_i \\ a \sum_{i=1}^n S_i t_i + b \sum_{i=1}^n t_i + c \sum_{i=1}^n t_i^2 &= \sum_{i=1}^n r_i t_i \end{aligned} \quad (10)$$

In matrix form, equation (10) can be expressed as:

$$\begin{pmatrix} \sum_{i=1}^n S_i^2 & \sum_{i=1}^n S_i & \sum_{i=1}^n S_i t_i \\ \sum_{i=1}^n S_i & n & \sum_{i=1}^n t_i \\ \sum_{i=1}^n S_i t_i & \sum_{i=1}^n t_i & \sum_{i=1}^n t_i^2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n r_i S_i \\ \sum_{i=1}^n r_i \\ \sum_{i=1}^n r_i t_i \end{pmatrix} \quad (11)$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n S_i^2 & \sum_{i=1}^n S_i & \sum_{i=1}^n S_i t_i \\ \sum_{i=1}^n S_i & n & \sum_{i=1}^n t_i \\ \sum_{i=1}^n S_i t_i & \sum_{i=1}^n t_i & \sum_{i=1}^n t_i^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^n r_i S_i \\ \sum_{i=1}^n r_i \\ \sum_{i=1}^n r_i t_i \end{pmatrix} \quad (12)$$

The solution of equation (12) gives the parameters of the model.

2.2. The Autoregressive (AR) Model

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t \quad (13)$$

Where β_0 and β_1 are the parameters of the model, ε_t is the error term with mean zero and the parameter constant variance. The β_0 and β_1 can be estimated by the method of least square. From equation 13

$$\varepsilon_t = Y_t - \beta_0 - \beta_1 Y_{t-1} \quad (14)$$

$$\varepsilon_t^2 = (Y_t - \beta_0 - \beta_1 Y_{t-1})^2 \quad (15)$$

Let

$$S = \sum_{t=1}^n \varepsilon_t^2 = \sum_{t=1}^n (Y_t - \beta_0 - \beta_1 Y_{t-1})^2 \quad (16)$$

Taking the derivatives of equation (16) with respect to β_0 and β_1 gives;

$$\frac{\partial S}{\partial \beta_0} = 2(-1) \sum_{t=1}^n (Y_t - \beta_0 - \beta_1 Y_{t-1}) \quad (17)$$

$$\frac{\partial S}{\partial \beta_1} = 2(-1) \sum_{t=1}^n ([Y_{t-1} (Y_t - \beta_0 - \beta_1 Y_{t-1})]) \quad (18)$$

$$\frac{\partial S}{\partial \beta_1} = \sum_{t=2}^n Y_{t-1} Y_t - \beta_0 \sum_{t=2}^n Y_{t-1} - \beta_1 \sum_{t=2}^n Y_{t-1}^2 \quad (19)$$

To obtain an equation for the parameters, set $\frac{\partial S}{\partial \beta_0}$ and $\frac{\partial S}{\partial \beta_1}$ to zero.

$$\sum_{t=1}^n Y_t - n\beta_0 - \beta_1 \sum_{t=2}^n Y_{t-1} = 0 \tag{20}$$

$$\sum_{t=2}^n Y_{t-1} Y_t - \beta_0 \sum_{t=2}^n Y_{t-1} - \beta_1 \sum_{t=2}^n Y_{t-1}^2 = 0 \tag{21}$$

$$n\beta_0 + \beta_1 \sum_{t=2}^n Y_{t-1} = \sum_{t=1}^n Y_t \tag{22}$$

$$\beta_0 \sum_{t=1}^n Y_{t-1} + \beta_1 \sum_{t=2}^n Y_{t-1}^2 = \sum_{t=1}^n Y_{t-1} Y_t \tag{23}$$

$$\begin{pmatrix} n & \sum_{t=2}^n y_{t-1} \\ \sum_{t=2}^n y_{t-1} & \sum_{t=2}^n y_{t-1}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} \sum_{t=1}^n y_t \\ \sum_{t=2}^n y_{t-1} y_t \end{pmatrix} \tag{24}$$

Let $p = \begin{pmatrix} n & \sum_{t=2}^n y_{t-1} \\ \sum_{t=2}^n y_{t-1} & \sum_{t=2}^n y_{t-1}^2 \end{pmatrix}$, $\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$ and

$$\gamma = \begin{pmatrix} \sum_{t=1}^n y_t \\ \sum_{t=2}^n y_{t-1} y_t \end{pmatrix}$$

$$p\beta = \gamma \tag{25}$$

$$\beta = p^{-1} \gamma \tag{26}$$

2.3. Statistical Tests for the Forecasting Models

2.3.1. Coefficient of Determination (r^2)

The coefficient of determination r^2 is used to determine the effectiveness of using the model in forecasting. It gives the coefficient of the total variance in the department variable explained by the model.

$$r^2 = \frac{\sum_{t=1}^n (Y_t - \bar{Y})^2}{\sum_{t=1}^n (Y_t - \bar{Y})^2} \tag{27}$$

Where, \hat{Y}_t and Y_t are the estimated and actual value of discharge.

Where n is the number of observation or data point.

2.3.2. Root Mean Square Error (RMSE)

$$RMSE = \frac{1}{n} \sum_{t=1}^n (Y_t - \hat{Y}_t)^2 \tag{28}$$

3. Results and Discussions

3.1. Harvey Model

Table 1. Results for Industrial Electricity Consumption Estimation using Harvey Model.

Parameter	Coefficient	Standard errors	r^2	RMSE
a	-0.5764	0.1632	0.80	65.2513
b	-24.346	10.4374		
c	0.01382	0.0055		

The Harvey Model is:

$$\ln \left(\frac{Y_t}{Y_{t-1}} \right) = -0.5764 \ln Y_{t-1} - 24.346 + 0.01382t \tag{29}$$

$$\ln \left(\frac{Y_t}{Y_{t-1}} \right) = -24.346 - 0.5764 \ln Y_{t-1} + 0.01382t \tag{30}$$

$$Y_t = Y_{t-1} e^{(-24.3446 - 0.5764 \ln Y_{t-1} + 0.01382t)} \tag{31}$$

From Table 1, with r^2 value of 0.80, it means that the Harvey model accounted for 80% of the variation in industrial electricity consumption. Moreover, The coefficient of t is positive (c = 0.01382) which means that industrial consumption increases with time. Table 2 and figure 1 present the actual and predicted value of industrial electricity consumption based on Harvey model.

Table 2. Actual and Predicted Industrial Electricity Consumption using Harvey Model.

Industrial electricity consumption (MW/h)				Industrial electricity consumption (MW/h)			
S/N	Year	Actual	Predicted	S/N	Year	Actual	Predicted
1	1979	160.3	169.98	19	1997	236.8	258.3
2	1980	199.7	173.55	20	1998	218.9	262.6
3	1981	121	193.15	21	1999	191.8	257.53
4	1982	262	158.35	22	2000	223.8	246.88
5	1983	254.4	222.79	23	2001	241.9	267.24
6	1984	217.2	223.09	24	2002	146.2	280.05
7	1985	259.8	211.53	25	2003	196	229.35
8	1986	280.5	231.39	26	2004	398	263.32
9	1987	294.1	242.36	27	2005	182.3	360.52

Industrial electricity consumption (MW/h)				Industrial electricity consumption (MW/h)			
S/N	Year	Actual	Predicted	S/N	Year	Actual	Predicted
10	1988	291.1	250.72	28	2006	383.44	262.51
11	1989	257.9	253.1	29	2007	494.01	364.81
12	1990	230.1	243.78	30	2008	421.6	411.84
13	1991	253.7	235.5	31	2009	428.954	390.43
14	1992	245.3	248.87	32	2010	395.591	398.77
15	1993	237.4	248.76	33	2011	426.37	390.68
16	1994	233.3	248.75	34	2012	457.92	408.9
17	1995	218.7	250.35	35	2013	518.7	427.33
18	1996	235.3	246.97	36	2014	594.48	456.79

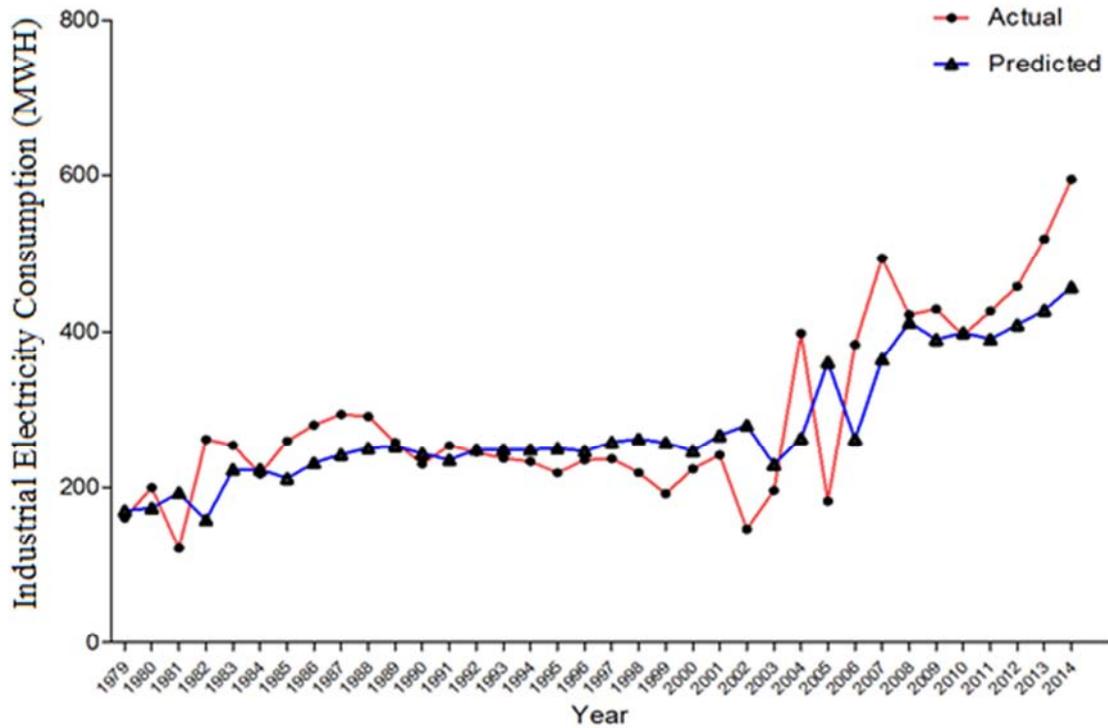


Figure 1. Actual and Predicted Industrial Electricity Consumption in Nigeria between 1979-2014.

3.2. Autoregressive Model

Table 3. Results for Industrial Electricity Consumption Estimation using Autoregressive model.

Variables	Coefficient	Standard Error	r ²	RMSE
ρ_0	76.4348	95.8456	0.501	71.3985
ρ_1	0.7552	0.1234		

RMSE = Root Mean Square Error.

Based on the model parameters shown in Table 3, the Autoregressive model for industrial electricity consumption is:

$$\hat{Y}_t = 76.4348 + 0.7552Y_{t-1} \tag{32}$$

Table 4. Actual and Predicted Industrial Electricity Consumption Using Autoregressive model.

Industrial Electricity Consumption (MW/h)				Industrial Electricity Consumption (MW/h)			
S/N	Year	Actual	Predicted	S/N	Year	Actual	Predicted
1	1979	160.3	195.53	19	1997	236.8	254.13
2	1980	199.7	197.49	20	1998	218.9	255.26

Where, \hat{Y}_t is the estimated industrial electricity consumption.

The model gave r² of 0.5010 which means that Autoregressive model was able to explained 50.10 percent of the variance in industrial electricity consumption (Table 3). The coefficient of Y_{t-1} is positive (0.7552) which means that industrial electricity consumption in Nigeria increases with time. The value of the actual and estimated industrial electricity consumption using the Autoregressive model is shown in Table 4 and figure 2.

Table 5 reveals that the Harvey model predicted better than the Autoregressive model as it gave higher value of coefficient of determination (r²=80.0%) and lower Root Mean Square Error (65.2513).

Industrial Electricity Consumption (MW/h)				Industrial Electricity Consumption (MW/h)			
S/N	Year	Actual	Predicted	S/N	Year	Actual	Predicted
3	1981	121	227.25	21	1999	191.8	241.75
4	1982	262	167.81	22	2000	223.8	221.28
5	1983	254.4	274.29	23	2001	241.9	245.45
6	1984	217.2	268.55	24	2002	146.2	259.11
7	1985	259.8	240.46	25	2003	196	186.84
8	1986	280.5	272.63	26	2004	398	224.45
9	1987	294.1	288.27	27	2005	182.3	377
10	1988	291.1	298.54	28	2006	383.438	214.11
11	1989	257.9	296.27	29	2007	494.01	366
12	1990	230.1	271.2	30	2008	421.6	449.51
13	1991	253.7	250.2	31	2009	428.954	394.82
14	1992	245.3	268.03	32	2010	395.591	400.38
15	1993	237.4	261.68	33	2011	426.37	375.18
16	1994	233.3	255.72	34	2012	457.92	398.42
17	1995	218.7	252.62	35	2013	518.7	422.25
18	1996	235.3	241.59	36	2014	594.48	468.15

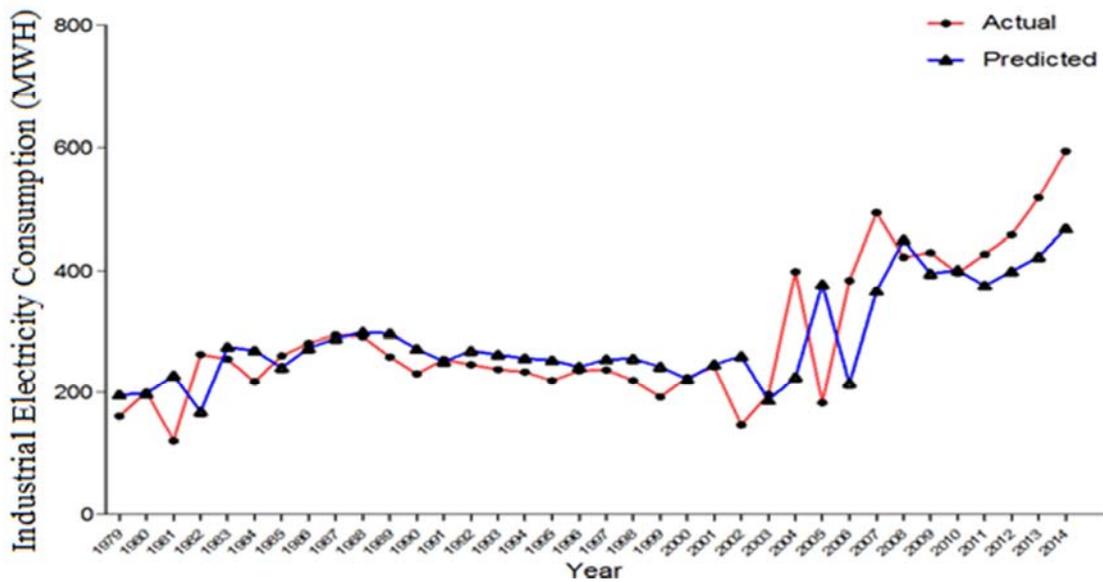


Figure 2. Actual and Predicted Industrial Electricity Consumption in Nigeria between 1979-2014 using Autoregressive model.

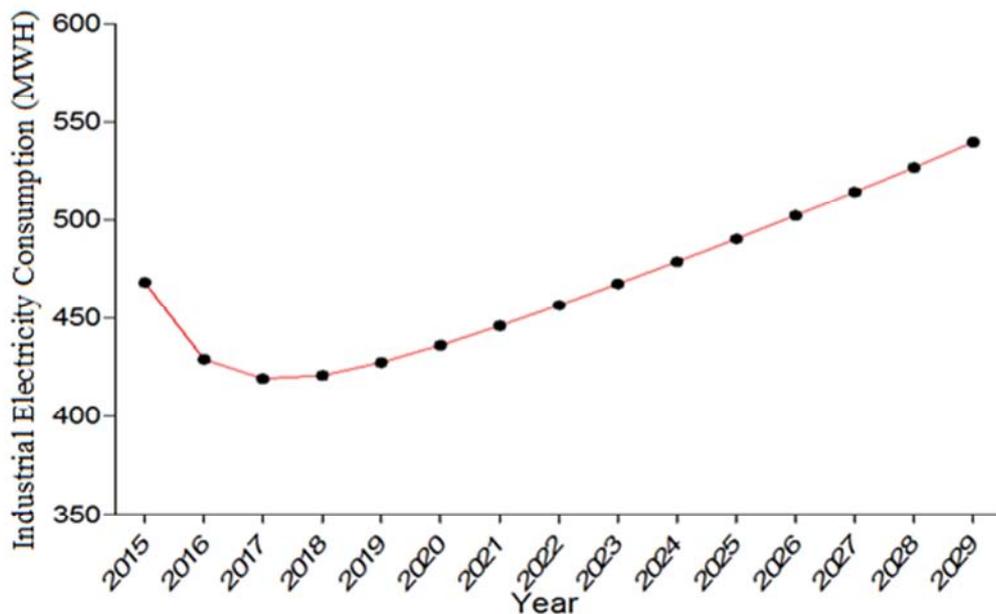


Figure 3. Graph of Forecasted Industrial Electricity Consumption in Nigeria (2015-2029).

Table 5. Comparison of the Forecasting Accuracy of the Harvey and Autoregressive model.

Models	r ² (%)	RMSE
Harvey	80.0	65.2513
Autoregressive	50.1	71.3985

3.3. Forecast of Industrial Electricity Consumption in Nigeria Using the Best Selected Model (Harvey Model)

Table 6. Forecast of Industrial Electricity Consumption using the Harvey Models (2015-2029).

S/N	Year	Forecast of industrial Electricity consumption (MW/h)
1	2015	468.18
2	2016	429.01
3	2017	419.17
4	2018	420.84
5	2019	427.4
6	2020	436.19
7	2021	446.08
8	2022	456.6
9	2023	467.53
10	2024	478.81
11	2025	490.39
12	2026	502.26
13	2027	514.42
14	2028	526.89
15	2029	539.65

The forecasting of electricity demand is obtained from the Harvey model by extrapolating the data from the year 2015 to 2029. The forecast values in Table 6 and figure 3 clearly indicate that the demand for electricity in Nigeria is continuously increasing. The industrial demand for electricity in Nigeria will increase from 468.18 MW/h in 2015 to 539.65 MW/h in 2029 which amounts to about 15.3 % increase.

4. Conclusion

Modeling and forecasting of industrial electricity consumption in Nigeria using two time series models is presented. The models are Harvey Model and Autoregressive Model. In the paper, data from 1979-2014 is use to predict the industrial electricity consumption and based on the results obtained Harvey model with better prediction accuracy is selected and used to forecast the industrial electricity consumption for the next fifteen years.

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